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## **Why $e^+e^- \rightarrow t\bar{t}$ is Different**

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# Why $e^+e^- \rightarrow t\bar{t}$ is different<sup>1</sup>

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## Abstract

Unlike other examples of fermion pair production in  $e^+e^-$  collisions, we show that top-quark pairs are produced in an essentially unique spin configuration in polarized  $e^+e^-$  colliders at all energies. Since the directions of the electroweak decay products of polarized top-quarks are strongly correlated to the top-quark spin axis, this unique spin configuration leads to a distinctive topology for top-quark pair events which can be used to constrain anomalous couplings to the top-quark. A significant interference effect between the *longitudinal* and *transverse* W-bosons in the decay of polarized top-quarks is also discussed.

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<sup>1</sup>Dedicated to Sid Drell on the occasion of the “Sid Drell Symposium” held at the Stanford Linear Accelerator Center, Stanford University, California on July 31, 1998.

At an  $\epsilon^+\epsilon^-$  collider with center of mass energy from 400 to 1000 GeV, top quark [1] pair production will be very different than fermion pair production at any previous  $\epsilon^+\epsilon^-$  machine for the following reasons: the photon ( $\gamma$ ),  $Z$ -boson and their interference will contribute approximately equally to the production and the top quark pairs will be produced at non-ultra-relativistic speeds. At present and previous  $\epsilon^+\epsilon^-$  colliders fermion pair production is either in the ultra-relativistic regime (LEP, SLC and TRISTAN) and/or completely dominated by the photon contribution (PETRA, PEP, CESR, SPEAR and CEA). This marked difference in the production mechanism leads to significantly different correlations, both angular and in spin, of the top and anti-top quarks compared with any previous fermion pairs produced in  $\epsilon^+\epsilon^-$  collisions. Moreover, even the spin correlations are measurable for top quark pair production because the top quark decays [2] before QCD effects de-correlate its spin and the decay products of a top quark are highly correlated with the direction of its spin *i.e.* *the top quark behaves more like a charged lepton than one of the light quarks in this regard.*

The matrix element for  $\epsilon_L^- \epsilon_R^+ \rightarrow t_s \bar{t}_{\bar{s}}$  is

$$\mathcal{M} \sim \frac{\epsilon^2}{S} \bar{\mathbf{v}}(\bar{\epsilon}) \gamma^\mu \gamma_L \mathbf{u}(\epsilon) \bar{\mathbf{u}}(t, s) \gamma_\mu \{ f_{LL} \gamma_L + f_{LR} \gamma_R \} \mathbf{v}(\bar{t}, \bar{s}) \quad (1)$$

where  $\gamma_{R,L} = \frac{(1 \pm \gamma_5)}{2}$ ,  $\sqrt{S}$  is the center of mass energy and  $s$  and  $\bar{s}$  are the spins of the top and anti-top quarks. The  $f_{IJ}$ 's are the sum of the photon and  $Z$ -boson products of couplings to the fermions corrected for the difference in the propagators. For top production the  $f_{IJ}$ 's have a very weak  $S$  dependence and are given by

$$f_{LL} = 1.22 - 1.19 \quad f_{LR} = 0.418 - 0.434 \quad \frac{f_{LR}}{f_{LL}} = 0.343 - 0.365 \quad (2)$$

from threshold to ultra-high energies. Without the  $Z$ -boson couplings,  $f_{LL} = f_{LR} = 2/3$  so that the contribution from the  $Z$ -boson is constructive for  $f_{LL}$  and destructive for  $f_{LR}$ . This difference between  $f_{LL}$  and  $f_{LR}$  and the fact top quark pairs are non-ultra-relativistic is what makes top pair production different than other examples of fermion pair production.

We will first consider the limit where  $f_{LR} = 0$ . In the ultra-high energy limit, where the mass of the top quark,  $m_t$ , can be neglected  $\sqrt{S} \gg 2m_t$ , the matrix element is

$$\mathcal{M} \sim \epsilon^2 f_{LL}(1 + \cos \theta^*), \quad (3)$$

where  $\theta^*$  is the scattering angle. In this limit the top quark is purely left-handed and the anti-top quark is purely right-handed,  $e_L^- e_R^+ \rightarrow t_L \bar{t}_R$ .

At threshold,  $\sqrt{S} \sim 2m_t$ , the matrix element is simply

$$\mathcal{M} \sim \epsilon^2 f_{LL} \quad (4)$$

and the direction of the top and anti-top quark spins are opposite the electron momentum or equivalently in the direction of the positron momentum.

At intermediate energies the matrix element interpolates between these two extremes

$$\mathcal{M} \sim \epsilon^2 f_{LL}(1 + \beta \cos \theta^*), \quad (5)$$

where  $\beta$  is the ZMF speed of the top quarks. What are the directions of the top and anti-top quark spins? In the rest frame of the top quark there are three natural possibilities for the direction of the top spin; the electron, the positron or the anti-top quark momentum directions. The threshold result excludes the anti-top quark momentum direction, which is undefined at threshold, leaving only the electron or positron momentum direction. Similarly the natural possibilities for the anti-top quark spin, in its rest frame, are the electron or positron momentum directions. The correct choice is *that the top quark spin vector is in the direction of the positron momentum in the top quark rest frame and the anti-top quark spin vector is in the direction opposite that of the electron momentum in the anti-top quark rest frame*. This spin basis has been called the beamline basis [3] and smoothly interpolates the required basis from threshold to ultra-high energies. An obvious question is, why is the spin of the top quark associated with the positron and the anti-top associated with the electron

instead of vice verse. If the term proportional to  $f_{LL}$  in the matrix element, eqn (1), is Fierz re-arranged one obtains

$$\sim f_{LL} [ \bar{\mathbf{u}}(t, s) \gamma_R \mathbf{v}(\bar{e}) ] [ \bar{\mathbf{u}}(e) \gamma_L \mathbf{v}(\bar{t}, \bar{s}) ]. \quad (6)$$

That is, the top quark spin is associated with the positron and anti-top quark spin with the electron<sup>2</sup>.

Returning to non-zero  $f_{LR}$ , the differential cross-sections obtained by explicit calculation, using the positron direction for the top quark spin and the electron direction for the anti-top quark spin in their respective rest frames, are

$$\begin{aligned} \frac{d\sigma}{d\cos\theta^*} (\epsilon_L^- \epsilon_R^+ \rightarrow t_\uparrow \bar{t}_\downarrow) &= \left( \frac{3\pi\alpha^2}{2S} \beta \right) \left[ f_{LL}(1 + \beta \cos\theta^*) + f_{LR} \frac{(1 - \beta^2)}{(1 + \beta \cos\theta^*)} \right]^2, \\ \frac{d\sigma}{d\cos\theta^*} (\epsilon_L^- \epsilon_R^+ \rightarrow t_\uparrow \bar{t}_\uparrow) &= \frac{d\sigma}{d\cos\theta^*} (\epsilon_L^- \epsilon_R^+ \rightarrow t_\downarrow \bar{t}_\downarrow) \\ &= \left( \frac{3\pi\alpha^2}{2S} \beta \right) f_{LR}^2 \frac{\beta^2(1 - \beta^2) \sin^2\theta^*}{(1 + \beta \cos\theta^*)^2}, \\ \frac{d\sigma}{d\cos\theta^*} (\epsilon_L^- \epsilon_R^+ \rightarrow t_\downarrow \bar{t}_\uparrow) &= \left( \frac{3\pi\alpha^2}{2S} \beta \right) f_{LR}^2 \frac{\beta^4 \sin^4\theta^*}{(1 + \beta \cos\theta^*)^2}. \end{aligned} \quad (7)$$

Note only the Up-Down spin configuration is non-zero if  $f_{LR} = 0$ , confirming the previous analysis. The Up-Up and Down-Down components are equal because CP is conserved both in the physics at this order of perturbation theory and in this spin basis. The  $\sin^2\theta^*$  factor in the Up-Up and Down-Down differential cross section implies that this is a P-wave and higher contribution. For the Down-Up component we have a  $\sin^4\theta^*$  indicative of a contribution starting at D-wave. At ultra-high energies only the Up-Down and Down-Up components are non-zero, giving the usual helicity basis result [4]. At threshold only the Up-Down component survives as expected.

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<sup>2</sup>If the  $f_{LR}$  term of the matrix element, eqn (1), was dominant then the top quark spin would be associated with the electron and the anti-top quark with the positron.

It should not be a surprise that by making small changes to the spin basis for small non-zero  $f_{LR}$ , the contributions from the two terms in the matrix element, eqn (1), can be made to totally destructively interfere for the Up-Up and Down-Down components. This spin basis has been called the Off-diagonal basis [5] and for small values of the ratio  $f_{LR}/f_{LL}$  gives the following differential cross sections;

$$\begin{aligned}
\frac{d\sigma}{d\cos\theta^*} (\epsilon_L^- \epsilon_R^+ \rightarrow t_\uparrow \bar{t}_\downarrow) &= \left( \frac{3\pi\alpha^2}{2S} \beta \right) f_{LL}^2 (1 + \beta \cos\theta^*)^2 (1 + \mathcal{O}(\frac{f_{LR}}{f_{LL}})) , \\
\frac{d\sigma}{d\cos\theta^*} (\epsilon_L^- \epsilon_R^+ \rightarrow t_\uparrow \bar{t}_\uparrow) &= \frac{d\sigma}{d\cos\theta^*} (\epsilon_L^- \epsilon_R^+ \rightarrow t_\downarrow \bar{t}_\downarrow) \equiv 0 , \\
\frac{d\sigma}{d\cos\theta^*} (\epsilon_L^- \epsilon_R^+ \rightarrow t_\downarrow \bar{t}_\uparrow) &= \left( \frac{3\pi\alpha^2}{2S} \beta \right) f_{LR}^2 \frac{\beta^4 \sin^4\theta^*}{(1 + \beta \cos\theta^*)^2} (1 + \mathcal{O}(\frac{f_{LR}}{f_{LL}})) .
\end{aligned} \tag{8}$$

The Down-Up component is suppress relative to the Up-Down component by  $(f_{LR}/f_{LL})^2$  as well as  $\beta^4$ . The factor  $(f_{LR}/f_{LL})^2$  gives an extra order of magnitude suppression to the Down-Up cross section for top production compared to examples where  $f_{LL} = f_{LR}$ . For a collider of  $\sqrt{s} = 400$  GeV the top quark pairs have a ZMF speed equal to 0.5c and the Up-Down component is 99.88% of the total cross section, see Fig. 1(a). In the helicity basis the LR component is only 52% of the total cross section at this energy, Fig. 2(a).

For  $\epsilon_L^+ \epsilon_R^-$  scattering<sup>3</sup> the same Off-diagonal spin basis can be used because  $f_{RL}/f_{RR} \approx f_{LR}/f_{LL}$  and similar results are obtained except that the dominant spin component is now Down-Up, see Fig. 1(b) and 2(b).

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<sup>3</sup> The couplings are  $f_{RR} = 0.882 - 0.868$   $f_{RL} = 0.185 - 0.217$   $f_{RL}/f_{RR} = 0.210 - 0.250$  from threshold to ultra-high energies respectively.

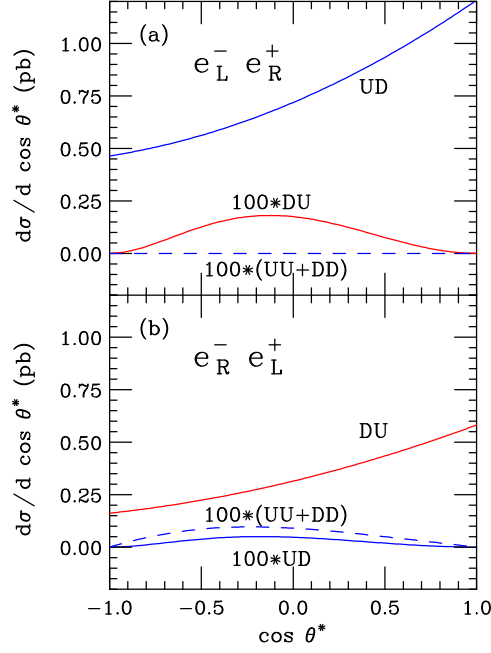


Figure 1: The spin configurations using the off-diagonal basis for both  $e_L^- e_R^+$  and  $e_R^- e_L^+$  for  $\sqrt{s} = 400 \text{ GeV}$ . Note that the sub-leading configurations have been amplified by a factor of 100 in these figures.

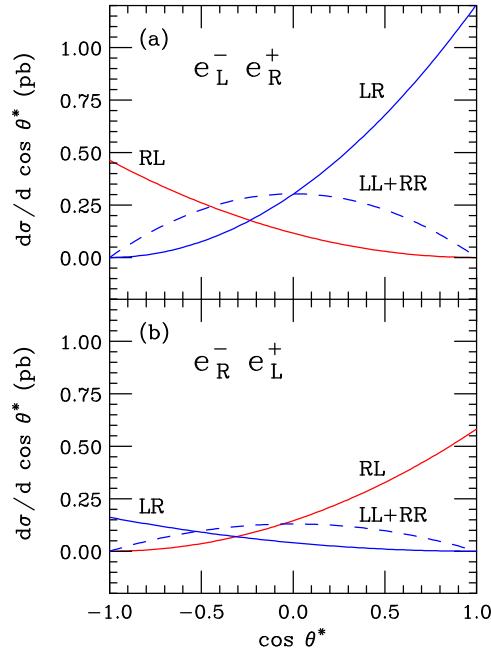


Figure 2: The spin configurations using the helicity basis for a  $\sqrt{s} = 400 \text{ GeV}$  collider.

What about the stability of this result under QCD radiative corrections? The quick answer is that the  $\mathcal{O}(\alpha_s)$  corrections are dominated by the soft gluon emission diagrams which factorize into an eikonal factor times the tree level result. That is, there is no spin flip from soft gluon emission and that only hard gluon emission or the  $\gamma/Z$  anomalous magnetic moment term can change the tree level result. An explicit calculation [6] of the  $\mathcal{O}(\alpha_s)$  corrections gives the ratio of the dominant top quark spin to the total as

$$\frac{\sigma(\epsilon_L^- \epsilon_R^+ \rightarrow t_\uparrow + X)}{\sigma(\epsilon_L^- \epsilon_R^+ \rightarrow t + X)} = 99.85\% \quad (9)$$

at  $\sqrt{s} = 400$  GeV. This is a small change from the tree level result of 99.88% even though the  $\mathcal{O}(\alpha_s)$  correction to the total cross section is 28% !

Since the top-quark pairs are produced in an unique spin configuration, and the electroweak decay products of polarized top-quarks are strongly correlated to the spin axis, the top-quark events at  $e^+e^-$  collider have a very distinctive topology. The predominant decay mode of the top-quark is  $t \rightarrow bW^+$ , with the  $W^+$  decaying either hadronically or leptonically. Let us first consider the single particle decay products correlations with the top-quark spin. If  $\chi_i^t$  is the angle between the top quark spin and the momentum of the  $i$ -th decay product measured in the top-quark rest-frame then the differential decay rate of the top-quark is

$$\frac{1}{\Gamma_T} \frac{d\Gamma}{d\cos\chi_i^t} = \frac{1}{2} [1 + \alpha_i \cos\chi_i^t], \quad (10)$$

where

$$\begin{aligned} \alpha_{e^+} &= \alpha_d \equiv 1 \\ \alpha_W &= -\alpha_b = \frac{m_t^2 - 2m_W^2}{m_t^2 + 2m_W^2} \approx 0.41 \\ \alpha_\nu &= \alpha_u \approx -0.31 \end{aligned}$$

see Jezabek and Kühn [7]. Fig. 3 shows these single particle correlations.

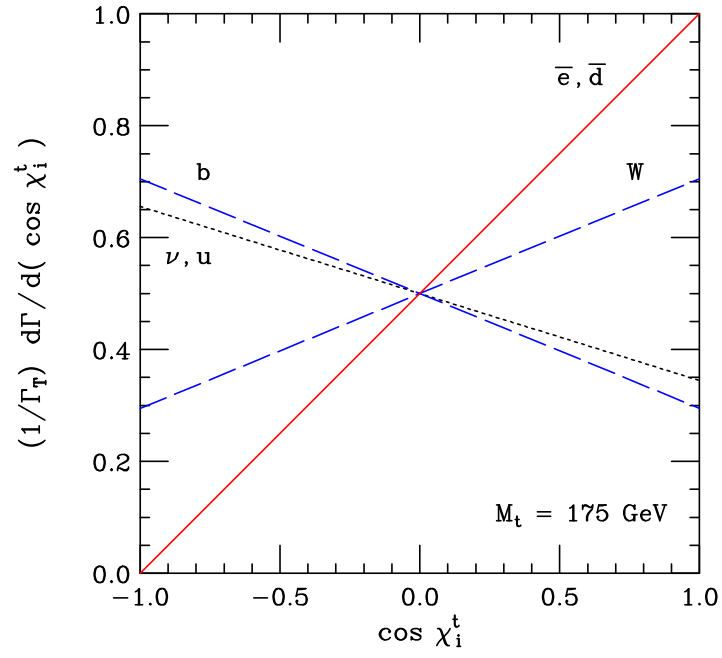


Figure 3: Correlations of the decay products of the top quark with the spin of the top quark.

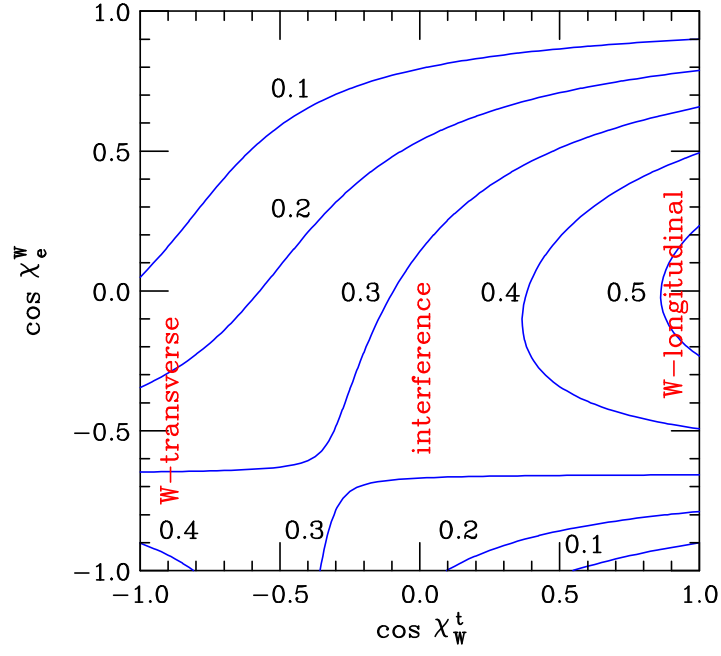


Figure 4: Contours of the top-quark differential angular decay distribution,  $\frac{1}{\Gamma_T} \frac{d^2 \Gamma}{d \cos \chi_e^w d \cos \chi_w^t}$ , in the  $\cos \chi_e^w - \cos \chi_w^t$  plane.

Note that the positron or d-quark are maximally correlated with the top quark spin. What is surprising about this is that the charged lepton or d-quark came from the decay of the  $W$ -boson which is less correlated with the spin of the top quark. This maximal correlation of the charged lepton or d-quark requires a subtle cancelation between the amplitudes involving transverse and longitudinal  $W$ -boson in Standard Model top quark decay[5].

Second, there are significant two particle correlations. Fig. 4 shows contour plots of the differential angular decay distribution in the  $\cos \chi_e^w$  verses  $\cos \chi_w^t$  plane where  $\pi - \chi_e^w$  is the angle between the direction of motion of the  $b$ -quark and the positron (or d-quark) in the  $W$ -boson rest-frame. When the  $W$ -boson momentum is parallel to the top spin,  $\cos \chi_w^t = 1$ , the  $W$ -boson is purely longitudinal as can be seen from the  $\cos \chi_e^w$  distribution of this figure along the right hand edge. Whereas, when the  $W$ -boson momentum is anti-parallel to the top spin,  $\cos \chi_w^t = -1$ , the  $W$ -bosons is purely transverse (left-handed). In between these two extremes both longitudinal and transverse  $W$ -boson contribute and there is significant interference effects between these two amplitudes.

In this paper we have shown that top quark pair production at an  $e^+e^-$  collider is very different than fermion pair production at current and past  $e^+e^-$  machines. At a polarized  $e^+e^-$  collider the top quark pairs are produced in an essentially unique spin configuration. In this configuration, the top-quark spin is strongly correlated with the positron spin direction determined in the top-quark rest-frame. The subsequent electroweak decays of the top-quark pair give decay products whose angular distributions are highly correlated with the parent top-quark spin. Top-quark pair events thus have a distinctive topology. This topology is sensitive to the top-quark couplings to the  $Z$ -boson and to the photon, which determine the orientation and the size of the top-quark and top anti-quark polarizations, as well as to the top-quark couplings to the  $W$  and the  $b$ -quark, which determine its decay distributions. Angular correlations in top-quark events may therefore be used to constrain deviations from the Standard Model. We have also shown that the interference between the *longitudinal* and

*transverse*  $W$ -bosons has a significant impact on the angular distribution of the top-quark decay products, and thus will provide additional means for testing the Standard Model predictions for top-quark decays.

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